

An Improved Fuzzy Predictive Control Algorithm and Its Application to an Industrial CSTR Process^{*}

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Abstract A finite horizon predictive control algorithm, which applies a saturated feedback control law as its local control law, is presented for nonlinear systems with time-delay subject to input constraints. In the algorithm, N free control moves, a saturated local control law and the terminal weighting matrices are solved by a minimization problem based on linear matrix inequality (LMI) constraints online. Compared with the algorithm with a nonsaturated local law, the presented algorithm improves the performances of the closed-loop systems such as feasibility and optimality. This model predictive control (MPC) algorithm is applied to an industrial continuous stirred tank reactor (CSTR) with explicit input constraint. The simulation results demonstrate that the presented algorithm is effective.

Keywords predictive control, fuzzy system, time-delay, saturated control law

1 INTRODUCTION

Model predictive control (MPC) has been widely accepted in industry for its ability to handle hard constraints on controls and states. It has, therefore, been widely used in petrochemical and related industries where satisfaction of constraints is particularly important because efficiency demands operating points on or close to the boundary of the set of admissible states and controls. Worst-case infinite horizon predictive control scheme has been studied extensively [1–3].

Time-delays usually exist in practice, especially in process industry. And time-delays can degrade the performances of closed-loop systems and even make the closed-loop systems unstable. Increasing attentions are paid to robust control for uncertain linear time-delay systems [4, 5]. MPC algorithms with a state feedback control law have been proposed [6–9] for uncertain systems with time-delay. A finite horizon state feedback predictive control algorithm for systems with time-delay has been presented [8]. The movement of N free control enlarges the feasible set of the system. But it must be solved by a nonlinear quadratic programming problem off-line.

Nonlinear systems always exist in practice and have been studied for a long time [10–12]. Takagi and Sugeno's (T-S) fuzzy model [13] has been widely used to represent complex nonlinear systems. Several fuzzy model-based predictive control algorithms [14, 15] have been studied, where the issue of transient control performance has been addressed as in general MPC. These strategies of fuzzy model predictive control are actually an extension of the conventional finite horizon MPC. However, their stability and feasibility cannot be explicitly addressed as those in infinite horizon MPC [1–3]. In this article, we extend the idea of worst-case infinite horizon predictive control into the

T-S model-based fuzzy predictive controller design.

A finite horizon predictive control algorithm is presented for nonlinear systems with time-delay, applying a saturated feedback control law [16–18] as its local control law. Compared with the algorithm based on nonsaturated local law, the presented algorithm has improved performances including feasibility and optimality. And also the algorithm is easy to be solved by a minimization problem based on linear matrix inequality (LMI) constraints on line. MPC algorithm is applied to an industrial continuous stirred tank reactor (CSTR) with explicit input constraints. The simulation results demonstrate that the presented algorithm is effective.

2 SYSTEM MODEL AND PRESENTATION

2.1 System description

A fuzzy dynamic model was proposed by Takagi and Sugeno [4] to represent local linear input/output relations of nonlinear systems. The fuzzy model is described by fuzzy IF-THEN rules. A T-S fuzzy model for nonlinear discrete-time system with time-delay is composed of r rules that can be represented as follows:

Rule l : If z_{k1} is M_{l1} ... and z_{kg} is M_{lg}

$$\text{Then } \begin{cases} \mathbf{x}_{k+1} = \mathbf{A}_l \mathbf{x}_k + \mathbf{A}_{dl} \mathbf{x}_{k-d} + \mathbf{B}_l \mathbf{u}_k, l = 1, 2, \dots, r \\ \mathbf{x}_k = \phi_k, k \in [-d, 0] \end{cases} \quad (1)$$

where $\mathbf{x} \in \mathbf{R}^{n_x}$ is the state, $\mathbf{u} \in \mathbf{R}^{n_u}$ is the control input, ϕ_k is an initial vector-valued function on $[-d, 0]$, d is a nonnegative integer denoting the known constant time-delay. r is the number of IF-THEN rules, $z_{k1}, \dots, z_{kg}$ is the premise variables, $M_{l1}, \dots, M_{lg}$ is the fuzzy set of the l th rule. $\mathbf{A}_l$, $\mathbf{A}_{dl}$ and $\mathbf{B}_l$ are known matrices with appropriate dimensions.

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At each time instant, the dynamic fuzzy model (1) can be described as the following global model:

$$\mathbf{x}_{k+1} = \mathbf{A}_k \mathbf{x}_k + \mathbf{A}_{dk} \mathbf{x}_{k-d} + \mathbf{B}_k \mathbf{u}_k \quad (2)$$

where $\mathbf{A}_k = \sum_{l=1}^r \theta_l(\mathbf{z}_k) \mathbf{A}_l$, $\mathbf{A}_{dk} = \sum_{l=1}^r \theta_l(\mathbf{z}_k) \mathbf{A}_{dl}$, $\mathbf{B}_k = \sum_{l=1}^r \theta_l(\mathbf{z}_k) \mathbf{B}_l$, $\theta_l(\mathbf{z}_k) = \omega_l(\mathbf{z}_k) / \sum_{p=1}^r \omega_p(\mathbf{z}_k)$, $\mathbf{z}_k = [z_{k1} z_{k2} \cdots z_{kg}]$, $\theta_l(\mathbf{z}_k) \geq 0$, $\sum_{l=1}^r \theta_l(\mathbf{z}_k) = 1$, $\omega_l(\mathbf{z}_k) = \prod_{q=1}^g M_{lq}(z_{kq})$, $M_{lq}(z_{kq})$ is the grade of membership of z_{kq} in the fuzzy set M_{lq} .

2.2 Problem formulation

The objective of the algorithm is to find the optimal control action subject to constraints:

$$\mathbf{U}_k = [\mathbf{u}_{k|k}^T \quad \mathbf{u}_{k+1|k}^T \quad \mathbf{u}_{k+2|k}^T \cdots] \quad (3)$$

and to minimize the worst case infinite horizon quadratic objective function at each time instant k :

$$\min_{\mathbf{u}_{k+i|k}, i \geq 0} J_k^\infty \quad (4)$$

$$\text{s.t. } J_k^\infty = \max_{[\mathbf{A}_{k+i} \mathbf{A}_{d(k+i)} \mathbf{B}_{k+i}]}_{i \geq 0} \sum_{i=0}^{\infty} [\|\mathbf{x}_{k+i|k}\|_Q^2 + \|\mathbf{u}_{k+i|k}\|_R^2] \quad (4a)$$

$$\begin{cases} \mathbf{x}_{k+i+1|k} = \mathbf{A}_{k+i} \mathbf{x}_{k+i|k} + \mathbf{A}_{d(k+i)} \mathbf{x}_{k+i-d|k} + \mathbf{B}_{k+i} \mathbf{u}_{k+i|k}, \mathbf{x}_{k|k} = \mathbf{x}_k \\ -\underline{\mathbf{u}} \leq \mathbf{u}_{k+i|k} \leq \bar{\mathbf{u}} \end{cases} \quad (4b)$$

where $\mathbf{Q}$ and $\mathbf{R}$ are the positive and symmetric weighting matrices. At time instant k , only the first computed control input $\mathbf{u}_{k|k}$ is implemented and the optimization problem (4) is repeated at time instant $k+1$ with new measurement.

2.3 Some preliminaries

For a matrix $\mathbf{P} \in \mathcal{R}^{n_u \times n_u} > 0$ and a positive number γ , an ellipsoid is defined as follows: $\varepsilon(\mathbf{P}, \gamma) = \{\mathbf{x} \in \mathcal{R}^{n_u} : \mathbf{x}^T \mathbf{P} \mathbf{x} \leq \gamma\}$. Also for a feedback gain matrix $\mathbf{F} \in \mathcal{R}^{n_u \times n_x}$, the set of states for which saturation does not occur is defined as: $\Pi(\mathbf{F}) = \{\mathbf{x} \in \mathcal{R}^{n_x} : |\mathbf{f}_m \mathbf{x}| \leq 1, m = 1, 2, \dots, n_u\}$, where $\mathbf{f}_m$ is the m th row of the matrix $\mathbf{F}$. The ellipsoid $\varepsilon(\mathbf{P}, \gamma)$ is inside $\Pi(\mathbf{F})$, i.e. $\varepsilon(\mathbf{P}, \gamma) \subset \Pi(\mathbf{F})$, if and only if $\gamma \mathbf{f}_m \mathbf{P}^{-1} \mathbf{f}_m^T \leq 1$, $m = 1, 2, \dots, n_u$.

Let $\mathcal{A}$ be the set of $n_u \times n_u$ diagonal matrices whose diagonal elements are either 1 or 0. There are 2^{n_u} elements in $\mathcal{A}$. Suppose that each element of $\mathcal{A}$ is labeled as $\mathbf{E}_j$, $j \in [1, 2^{n_u}]$ and denote $\mathbf{E}_j^- = \mathbf{I} - \mathbf{E}_j$. Clearly, $\mathbf{E}_j^- \in \mathcal{A}$.

Lemma 2.1 [17]: Let $\mathbf{F}_l, \mathbf{H}_l \in \mathcal{R}^{n_u \times n_x}$ be given, for an $\mathbf{x} \in \mathcal{R}^{n_x}$ if $\mathbf{x} \in \bigcap_{l=1}^L \Pi(\mathbf{H}_l)$, then

$$\text{sat}(\mathbf{F}\mathbf{x}) = \text{Co} \left\{ \left(\mathbf{E}_j \sum_{l=1}^L \theta_l \mathbf{F}_l + \mathbf{E}_j^- \sum_{l=1}^L \theta_l \mathbf{H}_l \right) \mathbf{x} : j \in [1, 2^{n_u}] \right\} \quad (5)$$

where Co denotes the convex hull, η_j is a state-dependent parameter satisfying $\eta_j \geq 0$, $\sum_{j=1}^{2^{n_u}} \eta_j = 1$.

Lemma 2.2 [19]: Let $\mathbf{G}$ be a matrix of full rank, $\mathbf{X}$ is a positive matrix, when $(\mathbf{G} - \mathbf{X})^T \mathbf{X}^{-1} (\mathbf{G} - \mathbf{X}) \geq 0$, the following inequality holds:

$$\mathbf{G}^T \mathbf{X}^{-1} \mathbf{G} \geq \mathbf{G}^T + \mathbf{G} - \mathbf{X} \quad (6)$$

Lemma 2.3 [20]: Define the set $\mathcal{S}_{d(k+i|k)}$ as $\mathcal{S}_{d(k+i|k)} = \text{Co}\{\mathbf{v}_{1(k+i|k)}, \dots, \mathbf{v}_{r^i(k+i|k)}\}$, $i > 0$. Assume that the state $\mathbf{x}_{k+i|k} \in \mathcal{S}_{d(k+i|k)}$, if $\mathbf{v}_{j(k+i+1|k)}$, $j = 1, 2, \dots, r^{i+1}$ satisfies:

$$\begin{aligned} \mathbf{v}_{j(k+i+1|k)} &= \mathbf{A}_l \mathbf{v}_{m(k+i|k)} + \mathbf{A}_{dl} \mathbf{v}_{n(k+i-d)} + \mathbf{B}_l \mathbf{u}_{k+i|k}, \\ l \in \{1, 2, \dots, r\}, \quad m &= 1, 2, \dots, r^i, \quad \begin{cases} n = 1, i-d \leq 0 \\ n = r^{i-d}, i-d > 0 \end{cases} \end{aligned} \quad (7)$$

Then $\mathbf{x}_{k+i+1|k} \in \mathcal{S}_{d(k+i+1|k)}$, $\mathcal{S}_{d(k+i+1|k)}$ is the tightest set that contains all possible states $\mathbf{x}_{k+i+1|k}$ of the system (2), and the set $\mathcal{S}_{d(k+i+1|k)}$ can be denoted as: $\mathcal{S}_{d(k+i+1|k)} = \text{Co}\{\mathbf{v}_{1(k+i+1|k)}, \dots, \mathbf{v}_{r^{i+1}(k+i+1|k)}\}$.

3 MAIN RESULTS

Theorem 3.1: For the fuzzy system with time-delay (2), N free control moves $[\mathbf{u}_{k|k}, \dots, \mathbf{u}_{k+N-1|k}]$ and a saturated feedback control law $\text{sat}(\mathbf{F}\mathbf{x}) =$

$$\text{Co} \left\{ \left(\mathbf{E}_j \sum_{p=1}^r \theta_p \mathbf{F}_p + \mathbf{E}_j^- \sum_{p=1}^r \theta_p \mathbf{H}_p \right) \mathbf{x} : j \in [1, 2^{n_u}] \right\}$$

minimize the worst case objective function (4) if there exist decision variables $\mathbf{u}_{k+i|k}$, $i \in [0, N-1]$, $\mathbf{G}, \mathbf{X}, \mathbf{Y}_p, \mathbf{Z}_p$, $p \in [1, r]$ such that the following convex minimization problem is feasible:

$$\lim_{\mathbf{u}_{k+i|k}, i \in [0, N-1], \mathbf{G}, \mathbf{X}, \mathbf{Y}_i, \mathbf{Z}_i, l \in [1, r], \mathbf{W}} \sum_{i=0}^N \gamma_i \quad (8)$$

$$\text{s.t.} \begin{bmatrix} \gamma_i & * & * \\ \mathbf{Q}^{1/2} \mathbf{v}_{j(k+i|k)} & \mathbf{I} & * \\ \mathbf{R}^{1/2} \mathbf{u}_{k+i|k} & 0 & \mathbf{I} \end{bmatrix} \geq 0, \quad \mathbf{v}_{j(k+i|k)} \in \text{vert}\{\mathbf{S}_{d(k+i|k)}\}, j \in [1, r^i], \quad i = 0, 1, \dots, N-1 \quad (8a)$$

$$\begin{bmatrix} 4\mathbf{X} & * & * & * & * & * \\ \left(\mathbf{A}_p \mathbf{G} + \mathbf{B}_p (\mathbf{E}_j \mathbf{Y}_q + \mathbf{E}_j^- \mathbf{Z}_q) + \mathbf{A}_q \mathbf{G} + \mathbf{B}_q (\mathbf{E}_j \mathbf{Y}_p + \mathbf{E}_j^- \mathbf{Z}_p) \right)^T & \mathbf{G}^T + \mathbf{G} - \mathbf{X} & * & * & * & * \\ \mathbf{W} (\mathbf{A}_{dp}^T + \mathbf{A}_{dq}^T) & 0 & \mathbf{W} & * & * & * \\ 0 & \mathbf{G} & 0 & \mathbf{W} & * & * \\ 0 & \mathbf{Q}^{1/2} / \mathbf{G} & 0 & 0 & \gamma_N \mathbf{I} & * \\ 0 & \mathbf{R}^{1/2} (\mathbf{E}_j \mathbf{Y}_p + \mathbf{E}_j^- \mathbf{Z}_p) & 0 & 0 & 0 & \gamma_N \mathbf{I} \end{bmatrix} > 0, \quad p \leq q \in [1, r], j \in [1, 2^{n_u}] \quad (8c)$$

$$\begin{cases} \bar{\mathbf{u}} - \mathbf{u}_{k+i|k} \geq 0 \\ \underline{\mathbf{u}} + \mathbf{u}_{k+i|k} \geq 0 \end{cases}, \quad i = 0, 1, \dots, N-1 \quad (8d)$$

$$\begin{bmatrix} 1 & \mathbf{z}_{p,m} \\ * & \mathbf{G} + \mathbf{G}^T - \mathbf{X} \end{bmatrix} > 0 \quad (8e)$$

where $\mathbf{z}_{p,m}$ is the m th row of $\mathbf{Z}_p$.

Proof: When the control action (3) is divided into two parts, the infinite horizon quadratic objective can also be divided into two parts:

$$\min_{\mathbf{u}_{k+i|k}} J_k^i, \quad i = 0, 1, \dots, N-1 \quad (9)$$

$$\min_{\mathbf{u}_{k+i|k}} \sum_{i=N}^{\infty} J_k^i, \quad i \geq N \quad (10)$$

with $J_k^i = \max_{[\mathbf{A}_{k+i}, \mathbf{A}_{d(k+i)}, \mathbf{B}_{k+i}] \in \Omega, i \geq 0} (\|\mathbf{x}_{k+i|k}\|_Q^2 + \|\mathbf{u}_{k+i|k}\|_R^2)$.

Because the predicted states lie in the convex set $\mathbf{S}_{d(k+i|k)}$, the upper bound of J_k^i , $j \in [0, N-1]$, can be obtained by evaluating the maximum of this term for each of the vertices of $\mathbf{S}_{d(k+i|k)}$. For $\forall \mathbf{x}_{k+i|k} \in \mathbf{S}_{d(k+i|k)}$, suppose that there exist $\gamma_i \in \mathfrak{R}$, $i = 0, \dots, N-1$ satisfying:

$$\|\mathbf{x}_{k+i|k}\|_Q^2 + \|\mathbf{u}_{k+i|k}\|_R^2 \leq \gamma_i, \quad i = 0, \dots, N-1 \quad (11)$$

γ_i is the upper bound of J_k^i . Apply Schur complementation, the inequality (11) is transformed into the LMI (8a).

The optimization problem (10) is an infinite horizon min-max optimization problem, so it can not be

$$\begin{bmatrix} 1 & \mathbf{v}_{j(k+N|k)}^T & * & \dots & * \\ \mathbf{v}_{j(k+N|k)} & \mathbf{X} & * & \dots & * \\ \mathbf{v}_{n_1(k+N-1)} & 0 & \mathbf{W} & \dots & * \\ \vdots & \vdots & \vdots & \ddots & * \\ \mathbf{v}_{n_d(k+N-d)} & 0 & 0 & \dots & \mathbf{W} \end{bmatrix} \quad (8b)$$

where $\mathbf{v}_{n_\tau(k+N-\tau)} \in \text{vert}(\mathbf{S}_{d(k+N-\tau)})$, $N-\tau \leq 0$, $n_\tau = 1$, $N-\tau > 0$, $n_\tau = r^{N-\tau}$, $\tau \in [1, d]$, $\mathbf{v}_{j(k+N|k)} \in \text{vert}\{\mathbf{S}_{d(k+N|k)}\}$, $j \in [1, r^N]$

$$\begin{bmatrix} * & * & * & * & * \\ * & * & * & * & * \\ 0 & \mathbf{W} & * & * & * \\ \mathbf{G} & 0 & \mathbf{W} & * & * \\ \mathbf{Q}^{1/2} / \mathbf{G} & 0 & 0 & \gamma_N \mathbf{I} & * \\ \mathbf{R}^{1/2} (\mathbf{E}_j \mathbf{Y}_p + \mathbf{E}_j^- \mathbf{Z}_p) & 0 & 0 & 0 & \gamma_N \mathbf{I} \end{bmatrix} > 0, \quad p \leq q \in [1, r], j \in [1, 2^{n_u}] \quad (8c)$$

solved explicitly. Define the following quadratic function:

$$V_{\text{LK}(k+i|k)} = \|\mathbf{x}_{k+i|k}\|_P^2 + \sum_{j=1}^d \|\mathbf{x}_{k+i-j|k}\|_S^2, \quad i \geq N \quad (12)$$

which satisfies:

$$V_{\text{LK}(k+i|k)} - V_{\text{LK}(k+i+1|k)} \leq -[\|\mathbf{x}_{k+i|k}\|_Q^2 + \|\mathbf{u}_{k+i|k}\|_R^2], \quad i \geq N \quad (13)$$

When the system is asymptotic stable, $\mathbf{x}_{\infty|k} = 0$, also $V_\infty = 0$. When (13) is summed from $i = 0$ to ∞ ,

$\sum_{i=N}^{\infty} J_k^i \leq V_{\text{LK}(k+N|k)}$. The upper bound of $\sum_{i=N}^{\infty} J_k^i$ can

not be computed directly. γ_N is the upper bound of

$\sum_{i=N}^{\infty} J_k^i$ when there exists $\gamma_N \in \mathfrak{R}$ satisfying:

$$\|\mathbf{x}_{k+N|k}\|_P^2 + \sum_{\tau=1}^d \|\mathbf{x}_{k+N-\tau|k}\|_S^2 \leq \gamma_N \quad (14)$$

The inequality (13) implies that $\Delta V_{k+i|k} = V_{k+i+1|k} - V_{k+i|k} < 0$, $\forall i \geq N$. If $V_{k+i|k} \leq \gamma_N$, then $V_{k+i+1|k} \leq \gamma_N$, $i \geq N$. Because $\mathbf{S} \geq 0$, $V_{k+i|k} \leq \gamma_N$, $i \geq N$ implies $\mathbf{x}_{k+i|k} \in \mathcal{E}(\mathbf{P}, \gamma)$, $i \geq N$, $\mathcal{E}(\mathbf{P}, \gamma) = \{\mathbf{x} \in \mathbf{R}^{n_x} | \mathbf{x}^T \mathbf{P} \mathbf{x} \leq \gamma_N\}$ is an invariant set. Let feedback gain $\mathbf{F}_p \in \mathfrak{R}^{n_u \times n_x}$,

$p \in [1, r]$ be given, if there exist $\mathbf{H}_p \in \mathfrak{H}^{n_u \times n_x}$, $p \in [1, r]$, such that $\varepsilon(\mathbf{P}, \gamma) \subset \bigcap_{p=1}^r \Pi(\mathbf{H}_p)$, then for the fuzzy system (2),

$$\text{sat}(\mathbf{F}\mathbf{x}) = \text{Co} \left\{ \left(\mathbf{E}_j \sum_{p=1}^r \theta_p \mathbf{F}_p + \mathbf{E}_j^- \sum_{p=1}^r \theta_p \mathbf{H}_p \right) \mathbf{x} : j \in [1, 2^{n_u}] \right\} \quad (15)$$

The closed-loop system is described as follows:

$$\begin{aligned} \mathbf{x}_{k+i+1|k} &= \mathbf{A}_{k+i} \mathbf{x}_{k+i|k} + \mathbf{A}_{d(k+i)} \mathbf{x}_{k+i-d|k} + \mathbf{B}_{k+i} \mathbf{u}_{k+i|k} \\ &= \sum_{p=1}^r \theta_p \left[\mathbf{A}_p \mathbf{x}_{k+i|k} + \mathbf{A}_{dp} \mathbf{x}_{k+i-d|k} + \mathbf{B}_p \sum_{q=1}^r \theta_q \sum_{j=1}^{2^{n_u}} \eta_j (\mathbf{E}_j \mathbf{F}_q + \mathbf{E}_j^- \mathbf{H}_q) \mathbf{x}_{k+i|k} \right] \\ &= \sum_{j=1}^{2^{n_u}} \eta_j \sum_{p=1}^r \theta_p \sum_{q=1}^r \theta_q \left\{ \left[\mathbf{A}_p + \mathbf{B}_p (\mathbf{E}_j \mathbf{F}_q + \mathbf{E}_j^- \mathbf{H}_q) \right] \mathbf{x}_{k+i|k} + \mathbf{A}_{dp} \mathbf{x}_{k+i-d|k} \right\} \\ &= \sum_{j=1}^{2^{n_u}} \eta_j \sum_{p=1}^r \theta_p \sum_{q=1}^r \theta_q \left[\left(\frac{\mathbf{E}_{j p q} + \mathbf{E}_{j q p}}{2} \right) \mathbf{x}_{k+i|k} + \frac{(\mathbf{A}_{dp} + \mathbf{A}_{dq})}{2} \mathbf{x}_{k+i-d|k} \right], \quad i \geq N \end{aligned}$$

where $\mathbf{E}_{j p q} = \mathbf{A}_p + \mathbf{B}_p \mathbf{A}_{j q}$, $\mathbf{A}_{j q} = \mathbf{E}_j \mathbf{F}_q + \mathbf{E}_j^- \mathbf{H}_q$. The inequality (13) becomes

$$\begin{aligned} &V_{k+i+1|k} - V_{k+i|k} + \left(\|\mathbf{x}_{k+i|k}\|_Q^2 + \|\text{sat}(\mathbf{u}_{k+i|k})\|_R^2 \right) \\ &= \left(\sum_{j=1}^{2^{n_u}} \eta_j \left(\sum_{p=1}^r \theta_p \sum_{q=1}^r \theta_q \left(\frac{\mathbf{E}_{j p q} + \mathbf{E}_{j q p}}{2} \mathbf{x}_{k+i|k} + \frac{\mathbf{A}_{dp} + \mathbf{A}_{dq}}{2} \mathbf{x}_{k+i-d|k} \right) \right) \right)^T \times \\ &\quad \mathbf{P} \left(\sum_{\rho=1}^{2^{n_u}} \eta_\rho \left(\sum_{s=1}^L \theta_s \sum_{t=1}^L \theta_t \left(\frac{\mathbf{E}_{j s t} + \mathbf{E}_{j t s}}{2} \mathbf{x}_{k+i|k} + \frac{\mathbf{A}_{ds} + \mathbf{A}_{dt}}{2} \mathbf{x}_{k+i-d|k} \right) \right) \right) + \\ &\quad \left(\sum_{j=1}^{2^{n_u}} \eta_j \sum_{p=1}^r \theta_p \mathbf{A}_{j p} \mathbf{x}_{k+i|k} \right)^T \mathbf{R} \left(\sum_{\rho=1}^{2^{n_u}} \eta_\rho \sum_{s=1}^L \theta_s \mathbf{A}_{\rho s} \mathbf{x}_{k+i|k} \right) - \|\mathbf{x}_{k+i|k}\|_P^2 + \|\mathbf{x}_{k+i|k}\|_S^2 - \|\mathbf{x}_{k+i-d|k}\|_S^2 + \|\mathbf{x}_{k+i|k}\|_Q^2 \\ &\leq \sum_{j=1}^{2^{n_u}} \eta_j \sum_{p=1}^r \theta_p \sum_{q=1}^r \theta_q \left\| \left(\frac{\mathbf{E}_{j p q} + \mathbf{E}_{j q p}}{2} \mathbf{x}_{k+i|k} + \frac{\mathbf{A}_{dp} + \mathbf{A}_{dq}}{2} \mathbf{x}_{k+i-d|k} \right) \right\|_P^2 + \\ &\quad \sum_{j=1}^{2^{n_u}} \eta_j \sum_{p=1}^r \theta_p \left\| \mathbf{A}_{j p} \mathbf{x}_{k+i|k} \right\|_R^2 - \|\mathbf{x}_{k+i|k}\|_P^2 + \|\mathbf{x}_{k+i|k}\|_S^2 - \|\mathbf{x}_{k+i-d|k}\|_S^2 + \|\mathbf{x}_{k+i|k}\|_Q^2 \\ &= \sum_{j=1}^{2^{n_u}} \eta_j \sum_{p=1}^r \theta_p \sum_{q=1}^r \theta_q \begin{bmatrix} \mathbf{x}_{k+i|k} \\ \mathbf{x}_{k+i-d|k} \end{bmatrix}^T \begin{bmatrix} \frac{\mathbf{E}_{j p q} + \mathbf{E}_{j q p}}{2} \mathbf{P} \frac{\mathbf{E}_{j p q} + \mathbf{E}_{j q p}}{2} - \mathbf{P} + \mathbf{S} + & * \\ \mathbf{Q} + \mathbf{A}_{j p}^T \mathbf{R} \mathbf{A}_{j p} & \\ \left(\frac{\mathbf{A}_{dp} + \mathbf{A}_{dq}}{2} \right)^T \mathbf{P} \frac{\mathbf{E}_{j p q} + \mathbf{E}_{j q p}}{2} & \left(\frac{\mathbf{A}_{dp} + \mathbf{A}_{dq}}{2} \right)^T \mathbf{P} \frac{\mathbf{A}_{dp} + \mathbf{A}_{dq}}{2} - \mathbf{S} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{k+i|k} \\ \mathbf{x}_{k+i-d|k} \end{bmatrix} \\ &= \sum_{j=1}^{2^{n_u}} \eta_j \sum_{p=1}^r \theta_p^2 \begin{bmatrix} \mathbf{x}_{k+i|k} \\ \mathbf{x}_{k+i-d|k} \end{bmatrix}^T \begin{bmatrix} \mathbf{E}_{j p p}^T \mathbf{P} \mathbf{E}_{j p p} - \mathbf{P} + \mathbf{S} + \mathbf{Q} + \mathbf{A}_{j p}^T \mathbf{R} \mathbf{A}_{j p} & * \\ \mathbf{A}_{dp}^T \mathbf{P} \mathbf{E}_{j p p} & \mathbf{A}_{dp}^T \mathbf{P} \mathbf{A}_{dp} - \mathbf{S} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{k+i|k} \\ \mathbf{x}_{k+i-d|k} \end{bmatrix} + \\ &\quad 2 \sum_{j=1}^{2^{n_u}} \eta_j \sum_{p,q=1, p < q}^r \theta_p \theta_q \begin{bmatrix} \mathbf{x}_{k+i|k} \\ \mathbf{x}_{k+i-d|k} \end{bmatrix}^T \begin{bmatrix} \frac{\mathbf{E}_{j p q} + \mathbf{E}_{j q p}}{2} \mathbf{P} \frac{\mathbf{E}_{j p q} + \mathbf{E}_{j q p}}{2} - \mathbf{P} + \mathbf{S} + & * \\ \mathbf{Q} + \mathbf{A}_{j p}^T \mathbf{R} \mathbf{A}_{j p} & \\ \left(\frac{\mathbf{A}_{dp} + \mathbf{A}_{dq}}{2} \right)^T \mathbf{P} \frac{\mathbf{E}_{j p q} + \mathbf{E}_{j q p}}{2} & \left(\frac{\mathbf{A}_{dp} + \mathbf{A}_{dq}}{2} \right)^T \mathbf{P} \frac{\mathbf{A}_{dp} + \mathbf{A}_{dq}}{2} - \mathbf{S} \end{bmatrix} \begin{bmatrix} \mathbf{x}_{k+i|k} \\ \mathbf{x}_{k+i-d|k} \end{bmatrix} \\ &\leq 0 \end{aligned}$$

When both of the two following inequalities are satisfied, the inequality (13) is satisfied.

$$\begin{bmatrix} \left(\frac{\mathcal{E}_{j pq} + \mathcal{E}_{j qp}}{2} \right)^T \mathbf{P} \frac{\mathcal{E}_{j pq} + \mathcal{E}_{j qp}}{2} - \mathbf{P} + \mathbf{S} + \mathbf{Q} + \mathbf{A}_{jp}^T \mathbf{R} \mathbf{A}_{jp} & * \\ \left(\frac{\mathbf{A}_{dp} + \mathbf{A}_{dq}}{2} \right)^T \mathbf{P} \frac{\mathcal{E}_{j pq} + \mathcal{E}_{j qp}}{2} & \left(\frac{\mathbf{A}_{dp} + \mathbf{A}_{dq}}{2} \right)^T \mathbf{P} \frac{\mathbf{A}_{dp} + \mathbf{A}_{dq}}{2} - \mathbf{S} \end{bmatrix} \leq 0, p \leq q \quad (16)$$

The inequality (16) is equivalent to the following inequality:

$$\begin{bmatrix} \mathbf{P} - \mathbf{S} - \mathbf{Q} - \mathbf{A}_{jp}^T \mathbf{R} \mathbf{A}_{jp} & * \\ 0 & \mathbf{S} \end{bmatrix} - \begin{bmatrix} \left(\frac{\mathcal{E}_{j pq} + \mathcal{E}_{j qp}}{2} \right)^T \\ \left(\frac{\mathbf{A}_{dp} + \mathbf{A}_{dq}}{2} \right)^T \end{bmatrix} \mathbf{P} \begin{bmatrix} \frac{\mathcal{E}_{j pq} + \mathcal{E}_{j qp}}{2} & \frac{\mathbf{A}_{dp} + \mathbf{A}_{dq}}{2} \end{bmatrix} \geq 0 \quad (17)$$

Apply Schur complementation, the above inequality becomes

$$\begin{bmatrix} 4\mathbf{P}^{-1} & * & * \\ \left(\mathcal{E}_{j pq} + \mathcal{E}_{j qp} \right)^T & \mathbf{P} - \mathbf{S} - \mathbf{Q} - \mathbf{A}_{jp}^T \mathbf{R} \mathbf{A}_{jp} & * \\ \left(\mathbf{A}_{dp} + \mathbf{A}_{dq} \right)^T & 0 & \mathbf{S} \end{bmatrix} \geq 0 \quad (18)$$

Define $\mathbf{X} = \gamma_N \mathbf{P}^{-1}$, $\mathbf{W} = \gamma_N \mathbf{S}^{-1}$, $\mathbf{Y}_p = \mathbf{F}_p \mathbf{G}$, $\mathbf{Z}_p = \mathbf{H}_p \mathbf{G}$ and $\mathbf{T}_1 = \text{diag}[\mathbf{I} \quad \mathbf{G} \quad \mathbf{W}]$. Multiply both sides of inequality (18) with γ_N , pre and pro multiply both of the sides of the inequality (18) with $\mathbf{T}_1^T$ and $\mathbf{T}_1$ respectively, apply Schur complementation and apply Lemma 2.2, the inequality (18) is transformed into the inequality (8c).

Because $\mathbf{x}_{k+N|k} \in \mathbf{S}_{d(k+N|k)}$, $\mathbf{x}_{k+N-\tau} \in \mathbf{S}_{d(k+N-\tau)}$, $\tau \in [1, d]$, $\mathbf{S}_{d(k+N|k)}$ and $\mathbf{S}_{d(k+N-\tau)}$ are convex sets, the inequality (14) is equivalent to

$$\mathbf{v}_{j(k+N|k)}^T \mathbf{P} \mathbf{v}_{j(k+N|k)} + \sum_{\tau=1}^d \mathbf{v}_{n_r(k+N-\tau|k)}^T \mathbf{S} \mathbf{v}_{n_r(k+N-\tau|k)} \leq \gamma_N \quad (19)$$

Apply Schur complementation, the inequality (19) becomes the LMI (8b).

Because the control action is divided into two parts, the control input constraints are considered as two parts. First consider the parameterized control moves $u_{k+i|k}$, $i \in [0, N-1]$ then the control input constraints are equivalent to the LMI (8d). When $i \geq N$, because $\mathbf{x}_{k+i|k} \in \varepsilon_P(\mathbf{P}, \gamma) \subset \Pi(\mathbf{H})$, $i \geq 0$, the input constraint $|\mathbf{f}_m^T \mathbf{x}_{k+i|k}| \leq 1$ is equivalent to $\mathbf{f}_m^T (\mathbf{P}/\gamma)^{-1} \mathbf{f}_m^T \leq 1$. Apply Schur complementation, the following inequality is satisfied:

$$\begin{bmatrix} 1 & \mathbf{f}_m \\ * & \gamma^{-1} \mathbf{P} \end{bmatrix} > 0 \quad (20)$$

Define $\mathbf{T}_2 = \text{diag}[\mathbf{I} \quad \mathbf{G}]$, pre and pro multiply both of the sides of the inequality (20) with $\mathbf{T}_2^T$ and $\mathbf{T}_2$ respectively, the inequality (20) is transformed into:

$$\begin{bmatrix} 1 & * \\ \mathbf{z}_{p,m}^T & \mathbf{G}^T \mathbf{X}^{-1} \mathbf{G} \end{bmatrix} > 0 \quad (21)$$

where $\mathbf{z}_{p,m} = \mathbf{f}_{p,m} \mathbf{G}$, $\mathbf{z}_{p,m}$ is the m th row of $\mathbf{Z}_p$. Apply the Lemma 2.2, the inequality can be transformed into the LMI (8e).

Theorem 3.2: When the optimization problem (8) is feasible at time instant k , the problem (8) is also feasible at time instant $k+1$, the receding optimization of the problem guarantees that the close-loop system is stable.

Proof: (1) Feasibility. In the optimization problem (8), only two inequalities (8a) and (8b) are related to the measured states $\mathbf{x}_{k|k}$, $\mathbf{x}_{k+N-\tau|k}$, $\tau \in [1, d]$ explicitly. Here it is necessary to prove that when the inequalities (8a) and (8b) are satisfied at time instant k , they must be satisfied at time instant $k+1$. $\mathbf{x}_{k|k} \in \mathbf{S}_{dk}$, $\mathbf{x}_{k+1|k+1} = \mathbf{A}_k \mathbf{x}_{k|k} + \mathbf{A}_{dk} \mathbf{x}_{k-d|k} + \mathbf{B}_k \mathbf{u}_{k|k}$, $\mathbf{x}_{k+1|k+1} \in \mathbf{S}_{d(k+1|k)}$. When the inequality (8a) is satisfied at time instant k , it must be satisfied at time instant $k+1$. It is satisfied at time instant $k+i$, $i \geq 2$ too.

From the inequality (8b), $\mathbf{x}_{k+N|k}^T \mathbf{P} \mathbf{x}_{k+N|k} +$

$\sum_{\tau=1}^d \mathbf{x}_{k+N-\tau|k}^T \mathbf{S} \mathbf{x}_{k+N-\tau|k} \leq \gamma_N$. The inequality (13) implies that $\mathbf{x}_{k+i+1|k}^T \mathbf{P} \mathbf{x}_{k+i+1|k} + \sum_{\tau=1}^d \mathbf{x}_{k+i+1-\tau|k}^T \mathbf{S} \mathbf{x}_{k+i+1-\tau|k} \leq$

$\mathbf{x}_{k+i|k}^T \mathbf{P} \mathbf{x}_{k+i|k} + \sum_{\tau=1}^d \mathbf{x}_{k+i-\tau|k}^T \mathbf{S} \mathbf{x}_{k+i-\tau|k}$, $i \geq N$, $\mathbf{x}_{k+i+1|k+1} \in \mathbf{S}_{d(k+i+1|k)}$, $\mathbf{x}_{k+i+1-\tau|k+1} \in \mathbf{S}_{d(k+i+1-\tau|k)}$. Then

$\mathbf{x}_{k+i+1|k+1}^T \mathbf{P} \mathbf{x}_{k+i+1|k+1} + \sum_{\tau=1}^d \mathbf{x}_{k+i+1-\tau|k+1}^T \mathbf{S} \mathbf{x}_{k+i+1-\tau|k+1} \leq \gamma_N$.

When the inequality (8b) is satisfied at time instant k , it must be satisfied at time instant $k+1$. It is satisfied at time instant $k+i$, $i \geq 2$ too.

(2) Stability. If the optimization problem (8) is feasible at time instant k , there exist optimal solutions $[\mathbf{u}_{k|k}^*, \mathbf{u}_{k+1|k}^*, \dots, \mathbf{u}_{k+N-1|k}^*]$, $\mathbf{G}^*$, $\mathbf{Y}^*$, $\mathbf{Z}^*$, $\mathbf{W}^*$ and optimal objective function values $\gamma_{k,0}^*, \dots, \gamma_{k,N}^*$, the feasible control moves can be selected as

$$\left[\mathbf{u}_{k+1|k}^*, \dots, \mathbf{u}_{k+N-1|k}^*, \text{sat} \left(\sum_{p=1}^r \theta_{kp} \mathbf{F}_{kp} \mathbf{x}_{k+N|k+1} \right) \right] \quad \text{with}$$

$\mathbf{F}_{kp} = \mathbf{Y}_p^* \mathbf{G}^{*-1}$ at time instant $k+1$. Suppose that the

objective function value is γ_{k+1} when the above feasible control moves are used, then

$$\gamma_k^* \geq \gamma_{k+1} + \gamma_{k,0}^* \quad (22)$$

Suppose that the optimal objective function value at time instant $k+1$ is γ_{k+1}^* , then

$$\gamma_{k+1} \geq \gamma_{k+1}^* \quad (23)$$

From the inequality (22) and (23), the following inequality is satisfied:

$$\gamma_k^* \geq \gamma_{k+1}^* \quad (24)$$

γ_k^* is shown to be a decreasing Lyapunov function of the closed-loop system, the closed-loop system is stable.

Algorithm 3.1: (1) initialize $\mathbf{x}_k$, $\mathbf{x}_{k-\tau}$, $\tau=1, \dots, d$, $\theta_l(z_k)$, $l \in \{1, 2, \dots, L\}$;

(2) at sampling time k , solve (8) and get the optimal variables γ^* , $\mathbf{G}^*$, $\mathbf{X}^*$, $\mathbf{Y}_p^*$, $\mathbf{Z}_p^*$, $p \in \{1, 2, \dots, L\}$, $\mathbf{W}^*$ and $\mathbf{u}_{k+i|k}^*$, $i \in [0, N-1]$;

(3) compute $\mathbf{x}_{k+1|k} = \mathbf{A}_k \mathbf{x}_{k|k} + \mathbf{A}_{dk} \mathbf{x}_{k-d|k} + \mathbf{B}_k \mathbf{u}_{k|k}^*$;

(4) set $k = k+1$, and go to the step (2).

4 CONTINUOUS STIRRED TANK REACTOR

Consider a CSTR in which the first-order irreversible exothermic reaction $A \rightarrow B$ occurs [4]. The reaction process is shown in the following figure:

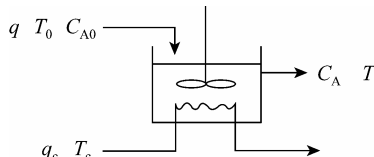


Figure 1 CSTR

The material and energy balance equations are

$$V \frac{dC_A}{dt} = \lambda q C_{A0} + q(1-\lambda)C_A(t-\alpha) - qC_A(t) - VK_0 \exp\left[\frac{-E}{RT(t)}\right] C_A(t)$$

$$Vc\rho \frac{dT}{dt} = qc\rho[\lambda T_0 + (1-\lambda)T(t-\alpha) - T(t)] +$$

$$V(-\Delta H)K_0 \exp\left[\frac{-E}{RT(t)}\right] C_A(t) - U(T(t) - T_w) \quad (25)$$

where $C_A(t) = \phi_1(t)$, $T(t) = \phi_2(t)$ for $t \in [-\alpha, 0]$. The objective is to control the concentration of A.

The above material and energy balance equations are reduced to dimensionless form using the notation:

$$x_1(t) = \frac{C_{A0} - C_A(t)}{C_{A0}}, \quad x_2(t) = \frac{T(t) - T_0}{T_0} \left(\frac{-E}{RT(t)} \right),$$

$$\theta_1(t) = \frac{C_{A0} - \phi_1(t)}{C_{A0}}, \quad \theta_2(t) = \frac{\phi_2(t) - T_0}{T_0} \left(\frac{-E}{RT(t)} \right),$$

$$t_{\text{new}} = \frac{t}{v}, \quad v = \frac{V}{q\lambda}, \quad \tau = \frac{\alpha}{v}, \quad \gamma_0 = \frac{E}{RT(t)},$$

$$D_a = K_0 v \exp(-\gamma_0), \quad \beta = \frac{Uv}{Vc\rho}, \quad H = \frac{(-\Delta H)C_{A0}}{c\rho T_0^2 R},$$

$$u(t) = \frac{T_0 - T_w}{T_0} \left(\frac{-E}{RT(t)} \right)$$

Define $\mathbf{x}(t) = [x_1(t) \quad x_2(t)]^T$, $f_1(x) = \frac{-1}{\lambda} x_1(t) + D_a[1 -$

$$x_1(t)] \exp\left(\frac{x_2(t)}{1+x_2(t)/\gamma_0}\right), \quad f_2(x) = -\left(\frac{1}{\lambda} + \beta\right) x_2(t) +$$

$$HD_a(1-x_1(t)) \exp\left(\frac{x_2(t)}{1+x_2(t)/\gamma_0}\right), \quad x_i(t) = \theta_i(t), \quad t \in$$

$[-\tau, 0]$, $i=1, 2$ then CSTR process can be described in the following dimensionless forms:

$$\dot{x}_1(t) = f_1(x) + \left(\frac{1}{\lambda} - 1\right) x_1(t-\tau),$$

$$\dot{x}_2(t) = f_2(x) + \left(\frac{1}{\lambda} - 1\right) x_2(t-\tau) + \beta u$$

and the linearization model of the above nonlinear system at the steady state point $(\mathbf{x}^s, \mathbf{u}^s)$ is given as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{x}^s, \mathbf{u}^s)(\mathbf{x}(t) - \mathbf{x}^s) + \mathbf{A}_d(\mathbf{x}^s, \mathbf{u}^s)(\mathbf{x}(t-\tau) - \mathbf{x}^s) + \mathbf{B}(\mathbf{x}^s, \mathbf{u}^s)(\mathbf{u} - \mathbf{u}^s), \quad \mathbf{x}^s = [x_1^s \quad x_2^s]^T$$

$$\mathbf{A}(\mathbf{x}^s, \mathbf{u}^s) = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_2}{\partial x} \end{bmatrix}^T (\mathbf{x}^s)$$

$$= \begin{bmatrix} \frac{1}{\lambda} - \frac{x_1^s}{1-x_1^s} & \frac{x_1^s}{(1+x_2^s/\gamma_0)^2} \\ \frac{Hx_1^s}{1-x_1^s} & -\left(\frac{1}{\lambda} + \beta\right) + \frac{Hx_1^s}{(1+x_2^s/\gamma_0)^2} \end{bmatrix},$$

$$\mathbf{A}_d(\mathbf{x}^s, \mathbf{u}^s) = \begin{bmatrix} \frac{1}{\lambda} - 1 & 0 \\ 0 & \frac{1}{\lambda} - 1 \end{bmatrix}, \quad \mathbf{B}(\mathbf{x}^s, \mathbf{u}^s) = \begin{bmatrix} 0 \\ \beta \end{bmatrix}.$$

Let the sampling time ΔT , the discrete model is given:

$$\mathbf{x}_{k+1} - \mathbf{x}^s = (\mathbf{A}(\mathbf{x}^s, \mathbf{u}^s)\Delta T + \mathbf{I})(\mathbf{x}_k - \mathbf{x}^s) + \mathbf{A}_d(\mathbf{x}^s, \mathbf{u}^s)\Delta T(\mathbf{x}_{k-d} - \mathbf{x}^s) + \mathbf{B}(\mathbf{x}^s, \mathbf{u}^s)\Delta T(\mathbf{u}_k - \mathbf{u}^s)$$

where $d = \tau/\Delta T$. There are three steady states when $u=0$, they are $x^1 = (0.1440, 0.8862)$, $x^2 = (0.4472, 2.7520)$, $x^3 = (0.7646, 4.7052)$. In this simulation, the

parameters are given: $\gamma_0 = 20$, $H = 8$, $\beta = 0.3$, $D_a = 0.072$, $\lambda = 0.8$, $\tau = 0.4$, $\Delta T = 0.2$. If the expected operating point $(\mathbf{x}_e, \mathbf{u}_e)$ is given, the T-S model of CSTR is described as follows.

Rule 1: if x_2 is about 0.8862, then

$$\bar{\mathbf{x}}_{k+1} = \mathbf{A}_1 \bar{\mathbf{x}}_k + \mathbf{A}_{d1} \bar{\mathbf{x}}_{k-d} + \mathbf{B}_1 \bar{\mathbf{u}}_k$$

Rule 2: if x_2 is about 2.7520, then

$$\bar{\mathbf{x}}_{k+1} = \mathbf{A}_2 \bar{\mathbf{x}}_k + \mathbf{A}_{d2} \bar{\mathbf{x}}_{k-d} + \mathbf{B}_2 \bar{\mathbf{u}}_k$$

Rule 3: if x_2 is about 4.7052, then

$$\bar{\mathbf{x}}_{k+1} = \mathbf{A}_3 \bar{\mathbf{x}}_k + \mathbf{A}_{d3} \bar{\mathbf{x}}_{k-d} + \mathbf{B}_3 \bar{\mathbf{u}}_k$$

where $\bar{\mathbf{x}}_k = \mathbf{x}_k - \mathbf{x}_e$, $\bar{\mathbf{x}}_{k-d} = \mathbf{x}_{k-d} - \mathbf{x}_e$, $\bar{\mathbf{u}}_k = \mathbf{u}_k - \mathbf{u}_e$

$$\mathbf{A}_1 = \begin{bmatrix} 0.7145 & 0.0151 \\ -0.2838 & 0.8112 \end{bmatrix}, \quad \mathbf{A}_2 = \begin{bmatrix} 0.5898 & 0.0792 \\ -1.2801 & 1.3234 \end{bmatrix},$$

$$\mathbf{A}_3 = \begin{bmatrix} 0.0944 & 0.0633 \\ -5.2446 & 1.1967 \end{bmatrix}$$

$$\mathbf{A}_{d1} = \mathbf{A}_{d2} = \mathbf{A}_{d3} = \begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix},$$

$$\mathbf{B}_1 = \mathbf{B}_2 = \mathbf{B}_3 = \begin{bmatrix} 0 \\ 0.06 \end{bmatrix}.$$

The membership functions are shown in Fig. 2.

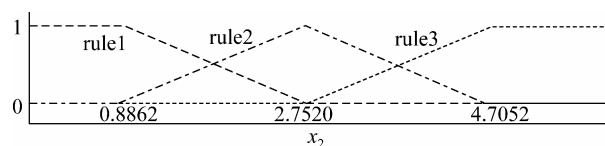


Figure 2 Membership functions

The weight matrices are $\mathbf{Q} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ and $\mathbf{R} = 0.02$. The control input satisfies the following constraint: $|u| \leq 2$. $\mathbf{x}_e = [0.4472; 2.7520]$ is an expected operating point. Here the presented algorithm is compared with the algorithm described in Ref. [9] and with the algorithm with non-saturated local control law. When the initial state is $\mathbf{x}_0 = [0.6; 3.4]$, the algorithm described in Ref [9] is infeasible, whereas the presented algorithm and the algorithm with nonsaturated local control law are feasible. Simulation results of both feasible algorithms are shown in Fig. 3, solid curves denote the results of the presented algorithm, and the dot-dashed curves denote that of the latter. From the Fig. 3, the closed-loop state responses can converge to the expected operating point, and the response speeds of the closed-loop system are similar; the control inputs satisfy the constraints, and the control input of the former can reach the saturating limit; the optimal

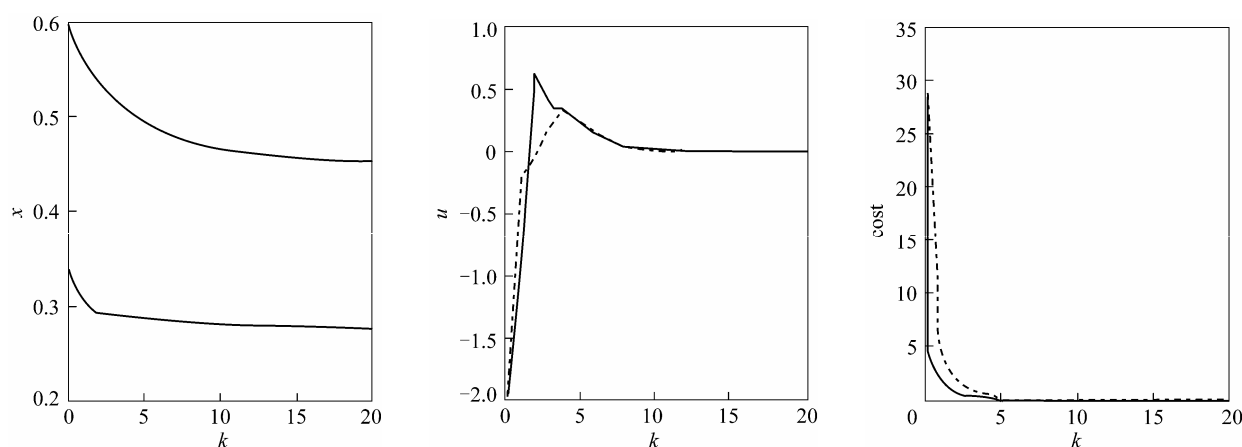


Figure 3 State responses, control input response and optimal objective function value $\mathbf{x}_0 = [0.6; 3.4]$

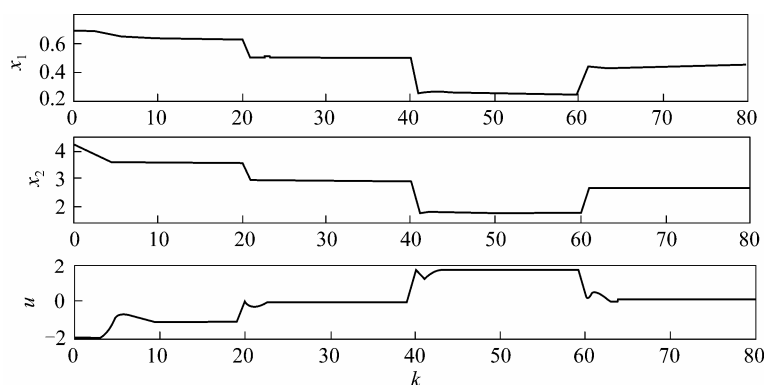


Figure 4 Closed-loop control performance

objective value of the former is evidently less than that of the latter and the optimality of the system is improved. When the initial state is $x_0 = [0.7; 4.2]$, the algorithm described in Ref. [9] and the algorithm with nonsaturated local control law are infeasible, whereas the presented algorithm is feasible. The presented algorithm has a larger initial feasible set. When the desired operating reference of the closed-loop system under stepwise changes from $[0.7; 4.2]$ to $[0.6420; 3.6050]$ to $[0.5012; 2.9450]$ to $[0.2503; 1.8120]$ and then to $[0.4472; 2.7520]$, the closed-loop state response *via* the presented algorithm follows the expected reference very well in Fig. 4.

5 CONCLUSIONS

An improved fuzzy predictive control algorithm applying saturated local control law is presented. It is a receding minimization problem based on LMI constraints. The stability of the presented algorithm is guaranteed as long as the minimization problem is solvable at the initial time instant. And the presented algorithm improves the performances of the controlled systems such as feasibility and optimality. The constrained CSTR problem is considered, the simulation results support the applicability of this algorithm to industrial problems.

NOMENCLATURE

$C_A(t)$	concentration of chemical A
C_{A0}	feed concentration
Co	convex hull
c	specific heat
E/R	ratio of Arrhenius activation energy to the gas constant
$-\Delta H$	heat of reaction (positive)
K_0	reaction velocity constant
n_x, n_u	dimension of x, u
q	feed flow rate
$T(t)$	reactor temperature
T_w	average coolant temperature in reactor cooling coil
T_0	feed temperature
U	heat transfer coefficient times the surface area of reactor
$u_{k+i k}$	control input at time $k+i$, computed at time k with $x_{k k} = x_k$
V	reactor volume
V_{LK}	Lyapunov-Krasovskii function
vert	vertex of the convex hull
$x_{k+i k}$	state at time $k+i$, predicted at time k
α	recycle delay time
λ	coefficient of recirculation satisfying $\lambda \in [0, 1]$
ρ	density

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